

Optimization of the 3rd Stage Rocket Trajectory Using Genetic Algorithm

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Abstract :- To optimize the third stage of a space launch vehicle, powered by Liquid Rocket Engine (LRE) and also to optimize the fuel efficiency by varying injection pressure and gravity turn. The space launch vehicle trajectory is designed analytically by using the general governing equations of the rocket. These trajectories are solved with the implementation of the genetic algorithm. The trajectories are designed and simulated with the commercial software MATLAB, furthermore the relation between parameters and generate MATLAB Coding to simulate the trajectory of the vehicle at 3rd stage. The governing equations are solved using Chebyshev polynomials subroutine and Lagrange polynomial equation available in MATLAB software. The variation of velocity, specific impulse, time is plotted for different parameter (injection pressure) values of the spacecraft.

A = reference area
 a_0 = reference sound speed
 a_T = thrust accelerate
 C_D = aerodynamic drag coefficient
 C_L = aerodynamic lift coefficient
D = specific drag
dt = integration time step
g = gravitational acceleration
 g_0 = surface gravitational acceleration
 G_h = gain on altitude rate term
 G_q = gain on dynamic pressure term
 H_{atmos} = atmospheric scale height
H = altitude
Isp = specific impulse
L = lift
q = dynamic pressure
R = radial distance to planet center
 r_0 = planetary surface radius
T = thrust
V = relative velocity
 Γ = flight path angle
 θ_0 = Initial Pitch angle
 ρ = local atmospheric density
 ρ_0 = reference density at surface
 σ = bank angle
 ψ = azimuth angle

ψ_0 = Thrust to weight ratio
q = pitch rate
 q_0 = constant pitch rate
 R_e = radius of the Earth

Subscripts:

LRE = Liquid Rocket Engine
LEO = Low Earth Orbit
GTO = Geo Transfer Orbit

1. Introduction

In this, we are going to discuss the advantages of our project. This shows the relationship between all parameters and based on this relation¹. It will be easy to carry required payload with fuel efficiency to any orbit (LEO, GEO). Here are few relations between the parameters they are the thrust depends on the diameter and injection pressure. The launch of spacecraft is fundamental to all space activity, and it is through our development of efficient launch vehicles that the immediate impact on space on many aspects of science, commerce, and daily life is possible². The launch lasts only a few minutes, and yet this short period, many years of development and investment in the commercial use of aircraft can be brought to nothing if one component of thousand components fails to perform to specification. In this we consider the forces acting on a space launcher and parameters required for space launcher powered by LRE.

Gravity turn or zero-lift turn is a maneuver used in launching a spacecraft into space. It is a trajectory optimization that uses gravity to steer the vehicle onto its desired trajectory. It offers two major benefits over a trajectory controlled solely through vehicle's thrust. The term gravity turn can also refer to the use of a planet's gravity to change a spacecraft's direction in other situations than entering or leaving altitude controller controls the orbit and altitude. as $\gamma = \gamma_0 - q_0(t - t_0)$. Genetic algorithms (GAs) are a subclass of evolutionary algorithms where the elements of the search space G are binary strings ($G = B^*$) or arrays of other elementary types⁵. The genotypes are used in the reproduction operations whereas the phenotypes compute the values of the objective functions $f \in F$ in the problem space X which are obtained via the genotype-phenotype

mapping "GPM." The roots of genetic algorithms go back to the mid-1950s, where biologists like Barricelli and the computer scientist Fraser began to apply computer-aided simulations to gain more insight into genetic processes and the natural evolution and selection.

2. Equation of Motion

Equations of motion are used to find out the parameters required for optimization of space launcher powered by LRE. The development of the equations of motion here is a modified form of those presented in (Wiesel, 1989: 216-219). With the state vector defined, the equations of motion can be developed further.

Using reference, we can make some observations. First, the vehicle stays on the H axis. Therefore the vertical acceleration is, and the downrange acceleration is represented as $\ddot{X}$. By geometry, the following equations can be produced.

The equation to find acceleration:

$$\frac{dv}{dt} = \frac{T - D}{m} - g_r \sin \beta$$

The equation to find direction:

$$\frac{d\beta}{dt} = -\frac{g_r}{v} \cos \beta$$

Gravity turn

Algorithm for solving gravity turn:

$$\frac{d\varphi}{dt} = -\frac{g_r}{v} \sin \varphi$$

$$\frac{dv}{dt} = \frac{T-D}{m} - g_r \sin \varphi \quad n = \frac{T-D}{mg_r}$$

Begin loop:

Let us take $z_0 = \tan \frac{\varphi_0}{2}$

$$c = \frac{v_0}{z_0^{n-1}} (1 + z_0^2)$$

$$\varphi = \varphi_0 + \Delta \varphi$$

$$Z = \tan \frac{\varphi}{2}$$

$$V = c z^{n-1} (1 + z^2)$$

$$\Delta t = \frac{c}{g_r} \left(z^{n-1} \left(\frac{1}{n-1} + \frac{z^2}{n+1} \right) - z_0^{n-1} \left(\frac{1}{n-1} + \frac{z_0^2}{n+1} \right) \right)$$

$$\Delta x = \frac{1}{2} (v_0 \sin \varphi_0 + v \sin \varphi) \Delta t$$

$$\Delta y = \frac{1}{2} (v_0 \cos \varphi_0 + v \cos \varphi) \Delta t$$

$$t = t_0 + \Delta t \quad x = x_0 + \Delta x \quad y = y_0 + \Delta y$$

$$m = m_0 - M \Delta t \Delta$$

update thrust lift and drag:

$$n = \frac{T-D}{mg_r}$$

new initial conditions:

$$z_0 = z, v_0 = v, \varphi_0 = \varphi, m_0 = m, t_0 = t$$

Repeat until we get the satisfactory results and end loop.

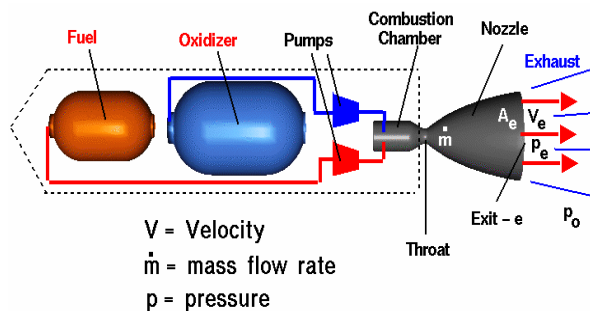
3. Liquid Propellant Rocket Engine

A liquid-propellant rocket uses liquid propellants. Liquids are desirable because their reasonably high density allows the volume of the propellant tanks to be relatively low, and it is possible to use lightweight centrifugal turbo pump to pump the propellant from the

tanks into the combustion chamber, which means that the propellants can be kept under low pressure. This permits the use of low-mass propellant tanks, resulting in a high mass ratio for the rocket.

An inert gas stored in a tank at a high pressure is sometimes used instead of pumps in simpler small engines to force the propellants into the combustion chamber. These engines may have a lower mass ratio, but are usually more reliable, therefore used widely in satellites for orbit maintenance.

Liquid rockets can be monopropellant rockets using a single type of propellant, bipropellant rockets using two types of propellant. Some designs are throttleable for variable thrust operation, and some may be restarted after a previous in-space shutdown. Liquid propellants are also used in hybrid rockets, in which a liquid oxidizer is combined with a solid fuel.



LIQUID PROPELLANT ROCKET ENGINE

PRINCIPLE OF OPERATION:

All liquid rocket engines have tankage and pipes to store and transfer propellant, an injector system, a combustion chamber which is very typically cylindrical, and one (sometimes two or more) rocket nozzles. Liquid systems enable higher specific impulse than solids and hybrid rocket engines and can provide very high tankage efficiency. Unlike gases, a typical liquid propellant has a density similar to water, approximately 0.7-1.4g/cm³ (except liquid hydrogen which has a much lower density), while requiring only relatively modest pressure to prevent vaporization. This combination of density and low pressure permits very lightweight tankage; approximately 1% of the contents for dense propellants and around 10% for liquid hydrogen (due to its low density and the mass of the required insulation).

For injection into the combustion chamber, the propellant pressure at the injectors needs to be greater than the chamber pressure; this can be achieved with a pump. Suitable pumps usually use centrifugal turbo pumps due to their high power and light weight, although reciprocating pumps have been employed in

the past. Turbo pumps are usually extremely lightweight and can give an excellent performance; with an on-Earth weight well under 1% of the thrust. Indeed, overall thrust to weight ratios including a turbo pump has been as high as 133:1 with the Soviet NK-33 rocket engine.

Alternatively, instead of pumps, a heavy tank of a high-pressure inert gas such as helium can be used, and the pump forwent; but the delta-v that the stage can achieve is often much lower due to the extra mass of the tankage, reducing performance; but for high altitude or vacuum use the tankage mass can be acceptable.

The major components of a rocket engine are therefore the combustion chamber (thrust chamber), pyrotechnic igniter, propellant feed system, valves, regulators, the propellant tanks, and the rocket engine nozzle. Regarding feeding propellants to the combustion chamber, liquid-propellant engines are either pressure-fed or pump-fed, and pump-fed engines work in either a gas-generator cycle, a staged-combustion cycle, or an expander cycle.

A liquid rocket engine (LRE) can be tested before use, whereas for a solid rocket motor a rigorous quality management must be applied during manufacturing to ensure high reliability. An LRE can also usually be reused for several flights, as in the Space Shuttle.

4. Chebyshev polynomials

The Chebyshev polynomials T_n or U_n are polynomials of degree n and order of Chebyshev polynomials of either kind composes a polynomial order. The Chebyshev polynomials T_n are polynomials with the largest possible leading coefficient, but subject to the condition that their absolute value on the interval $[-1,1]$ is bounded by 1³. They are also the extremal polynomials for many other properties.

The recurrence relation defines the Chebyshev polynomials of the first kind

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) \\ \sum_{n=0}^{\infty} T_n(x)t^n &= \frac{1-tx}{1-2tx+t^2} \\ \sum_{n=1}^{\infty} T_n(x)\frac{t^n}{n} &= \ln \frac{1}{\sqrt{1-2tx+t^2}} \end{aligned}$$

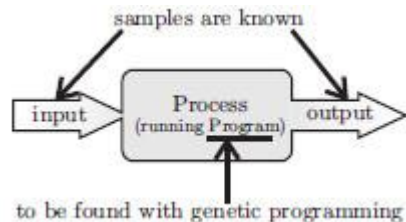
The basic Chebyshev equation is ⁴

$$F(x) = \sum_{n=0}^{\infty} a_n T_n(x)$$

5. Genetic algorithm:

The term Genetic Programming¹ (GP) has two possible meanings. Firstly it is often used to subsume

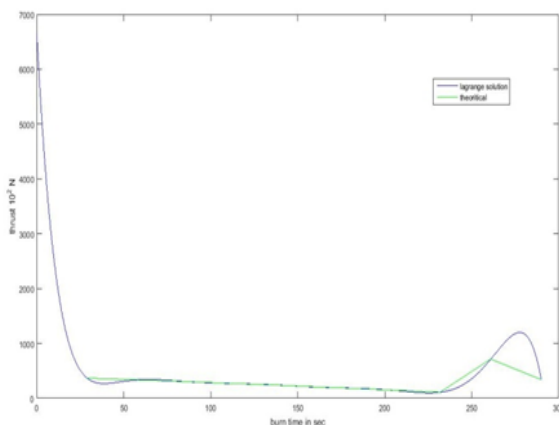
all evolutionary algorithms that have tree data structures as genotypes. Secondly, it can also be defined as the set of all evolutionary algorithms that breed programs², algorithms, and similar constructs. In this, we focus on the latter definition which still includes discussing tree-shaped genomes⁶. The conventional well-known input-processing-output model³ from computer science states that a running instance of a program uses its input information to compute and return output data. In Genetic Programming⁴, usually, some inputs or situations and corresponding output data samples are known or can be produced or simulated. The goal then is to find a program that connects them or that exhibits some desired behavior according to the specified situations, as sketched Process



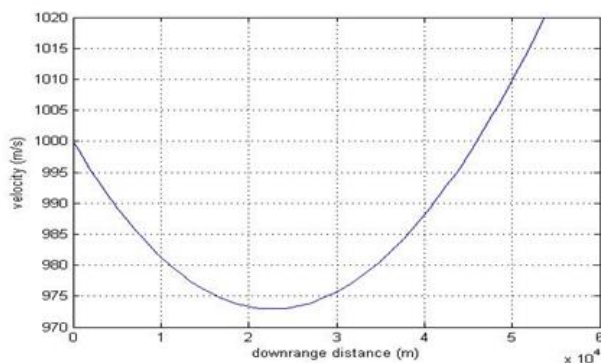
The structure of the Genetic algorithm.

6. Numerical solutions:

Time vs. Thrust for segmented trajectory



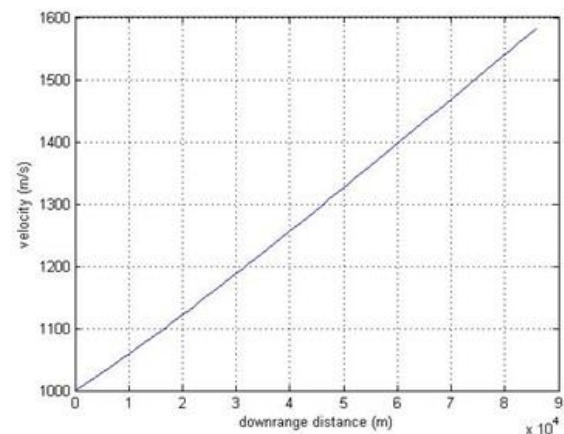
Velocity variation with respect to range.



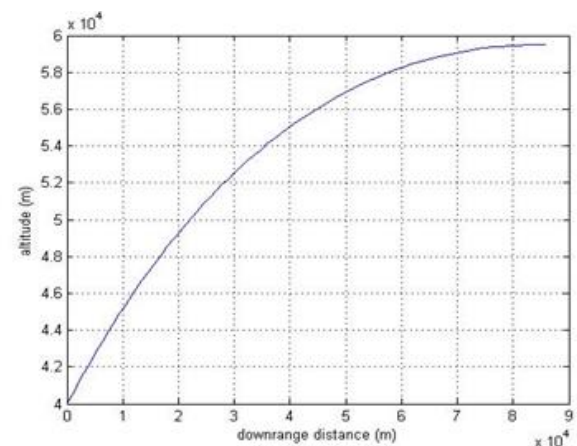
Mass change with respect to downrange distance



Velocity variation with respect to range

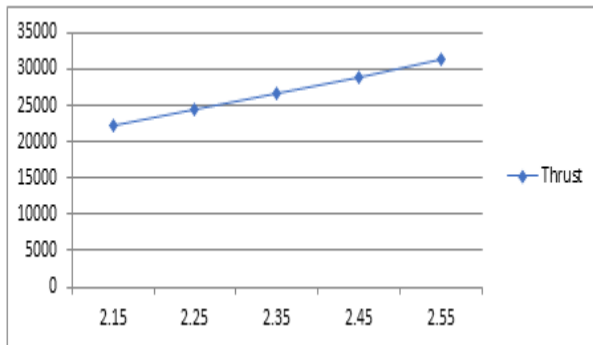


Altitude change with respect to range

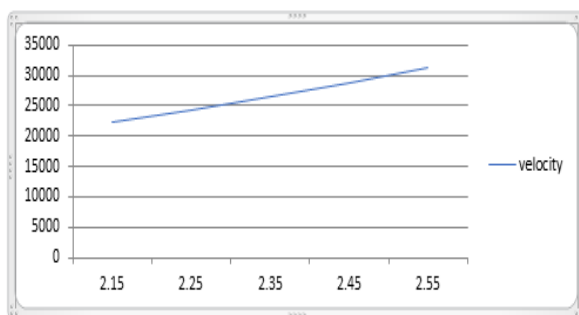


7. Genetic algorithm solutions

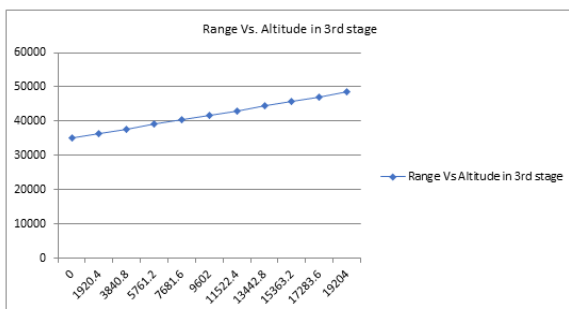
Specific impulse vs. thrust



Specific impulse vs. velocity



Trajectory of 3rd stage using gravity turn algorithm



8. Conclusion:

This paper presents the optimized trajectory for the rocket using numerical techniques and genetic algorithm. The trajectory is optimized through the segmented values obtained from thrust. At each descriptive node the values of specific impulse is found from the injection pressure. The gravity turn of the rocket is also found in this paper which is considered as the main factor for the rocket to launch in an orbit. The trajectory is optimized for different parameters.

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