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Reverse Super Edge-Magic Labeling on W-trees

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Abstract:- A reverse edge-magic labeling of a graph G is a one-to-one map f from V(G) ∪E(G) on to the integers {1,2,3,…., $|V(G)\cup E(G)|$} with the property that , there is an integer constant c such that f(xy)-{f(x)+f(y)}=c for any (x,y) ∈ E(G) . If f (V(G)) = {1,2,3,…$|V(G)|$} then the reverse edge – magic labeling is called reverse super edge-magic labeling . In this paper we formulate reverse super edge-magic labeling on w-trees.

Index Terms- Reverse super edge-magic labeling, w-graph, and w-tree.

1. Introduction

All graphs in this paper are finite, simple, planar and undirected. The graph G has the vertex set V (G) and edge-set E (G). A general reference for graph-theoretic ideas can be seen in [11].

A labeling of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper the domain will be the set of all vertices and edges and such labeling are called total labeling. Some labelings use the vertex-set only, or the edge-set only and we shall call them as vertex-labeling and edge-labelings respectively. Other domains are possible. There are many types of graph labeling, for example harmonious, cordial, and graceful and anti-magic.

In this paper, we focus on one type of labeling called reverse edge-magic labeling. A reverse edge-magic labeling of a graph G is a one-to-one map f from V(G) ∪E(G) on to the integers {1,2,3,…., $|V(G)\cup E(G)|$} with the property that , there is an integer constant c such that f(xy)-{f(x)+f(y)}=c for any (x,y) ∈ E(G) . If f(V(G)) = {1,2,3,…$|V(G)|$} then the reverse edge – magic labeling is called reverse super edge-magic labeling .A number of classification studies on reverse super edge-magic graphs has been intensively investigated.

Definition 1: Let G be a graph with set of vertices and edges as

V(G)= $\{(c_1,c_2,b,w,d)\cup(x^1,x^2,\ldots,x^n)\cup(y^1,y^2,\ldots,y^n)\}$

E(G)= $\{(c_1x^1,c_1x^2,\ldots,c_1x^n)\cup(c_2y^1,c_2y^2,\ldots,c_2y^n)\cup(c_1b,c_1w)\cup(c_2w,c_2d)\}$

.

We shall call it w- graph and it shall be denoted by w (n).

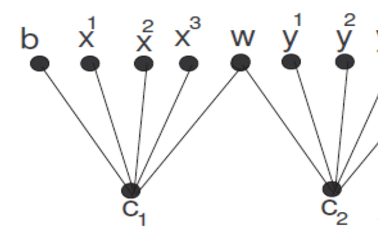

Fig. 1. $W(3)$

Definition 2:- Suppose that we have k isomorphic copies of w-graphs W (n). A w-tree WT (n, k) is a tree obtained by taking a new vertex α and joining it with $\{d_i : 1\le i\le k\}, where$ n≥2 and k≥3.

In this paper, we present reverse super edge-magic labeling for w-graph W (n) and w-tree WT (n, k).

2. Main Results

Before giving our main results, let us consider the following lemma which gives a necessary and sufficient condition for a graph to be reversed super edge-magic.

Lemma 1:- A graph G with v vertices and e edges is reverse super edge-magic if and only if there exists a bijective function f: V(G) → $\{1,2,3,\ldots,v\}$ such that the set S={f(x)+f(y)/xy∈E(G)} consists of e consecutive integers . In such a case, f extends to a reverse super edge-magic labeling of G with magic constant c = v + e –s where s = max(S) and

$S = \{f(x) + f(y)/xy \in E(G)\} = \{(v+1) - c, (v+2) - c, \ldots$

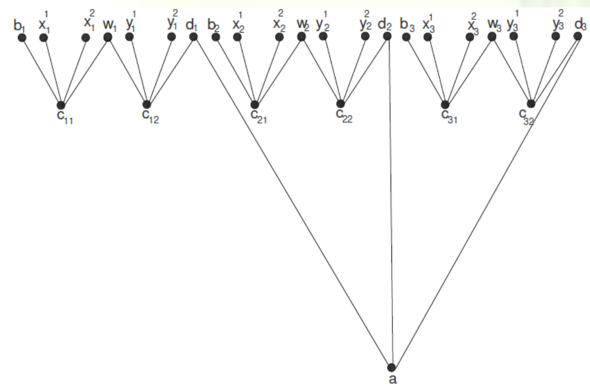

Fig. 2. w-tree $WT(2,3)$

In the following Theorem we develop reverse super edge-magic labeling of w- trees.

Theorem1:- $G \cong W(n)$ admits reverse super edge-magic labeling if $n \geq 1$

Proof:- Let $v = |V(G)|$, $e = |E(G)|$ then v = 2n+5 and $e = 2n + 4$. We denote the vertex and edge sets of G as follows:

V (G)=$\{(c_1, c_2, b, w, d) \cup (x^1, x^2, \ldots, x^n) \cup (y^1, y^2, \ldots, y^n)\}$

E(G)=

$\{(c_1x^1, c_1x^2, \ldots, c_1x^n) \cup (c_2y^1, c_2y^2, \ldots, c_2y^n) \cup (c_1b, c_1w) \cup (c_2w, c_2d)\}$

.

Now, we define the labeling f :V→{1,2,3,…,v} as follows

f(b)= 1 , $f(w) = n + 2$ f(d)= 2n+3

$f(c_1) = 2n + 6 f(c_2) = 2n + 7$

$f(x^i) = i + 1$,$1 \leq i \leq n$ and

$f(y^j) = n + j + 2,\quad 1 \leq j \leq n$

The set of all edge- sums generated by the above formula forms a consecutive integer sequence 2n+6, 2n+7…, 4n+8. Therefore by Lemma 1, f can be extended to reverse super edge-magic labeling and we obtain the magic constant

$c = v + e - s$

$c = 2n + 5 + 2n + 4 - \{4n + 8\}$

= 1

For example, here we show the W-graphW (3).

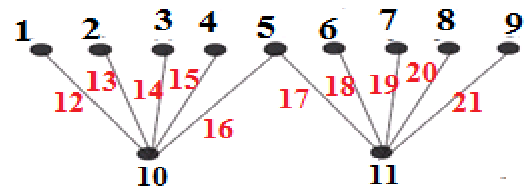

Fig 3 : WT(3)

Theorem2:- For $k \geq 3$, $G \cong WT(n,k)$ admits reverse super edge-magic labeling if $n \geq k - 1$

For even k and $n \geq k$ for odd k.

Proof:- Let $v = |V(G)|$, $e = |E(G)|$ then $v = (2n+3)k + 2k + 1$ and $e = (2n+5)k$. We denote the vertex and edge sets of G as follows:

$V(G) = \{a\} \cup \{b_i : 1 \leq i \leq k\} \cup \{w_i : 1 \leq i \leq k\} \cup \{d_i : 1 \leq i \leq k\} \cup \{c_{i1} : 1 \leq$
$\cup \{x_i^l : 1 \leq i \leq k, 1 \leq l \leq n\} \cup \{y_i^l : 1 \leq i \leq k, 1 \leq l \leq n\}$

$E(G) = \{b_ic_{i1} : 1 \leq i \leq k\} \cup \{w_ic_{i1} : 1 \leq i \leq k\} \cup \{w_ic_{i2} : 1 \leq i \leq k\} \cup \{d_ic_{i2}$
$\{ad_i : 1 \leq i \leq k\} \cup \{c_{i1}x_i^l : 1 \leq i \leq k, 1 \leq l \leq n\} \cup \{c_{i1}y_i^l : 1 \leq i \leq k, 1 \leq l \leq$

Before formulating labeling, let s = $\frac{k}{2}$ for even k and$s = \frac{k-1}{2} for odd k$.

Now, we define the labeling f: V→{1,2,3,…,v} as follows

$f(a) = v - 2\left\lceil\frac{k}{2}\right\rceil$

$f(w_i) = (2n+3)i - n - 1, 1 \leq i \leq k$

$f(c_{i1}) = \begin{cases} (2n+3)k + 2i - 1, & for\ 1 \leq i \leq s \\ (2n+3)k + 2i + 1, & for s+1 \leq i \leq k \end{cases}$

$f(c_{i2}) = (2n+3)k + 2i\quad ,1 \leq i \leq k$

$f(b_i) = \begin{cases} (2n+3)i - 2n - 2, & for\ 1 \leq i \leq s \\ (2n+3)i & , for s+1 \leq i \leq k \end{cases}$

$f(d_i) = \begin{cases} f(w_i) + 2i + n - 2s + 1 & ,for\ 1 \leq i \leq s \\ f(w_i) + 2i - n - 2s - 3, & for s+1 \leq i \leq k \end{cases}$

For 1≤l≤n,

$f(x_i^l) =$

$\begin{cases} (2n+3)i - 2n + l - 2, & if\ 1 \leq i \leq s \\ (2n+3)i - n + l - 1 & ,if s+1 \leq i \leq k \end{cases}$

For $1 \le i \le s$,

$$f(y_i^l) = \begin{cases} (2n+3)i - n + l - 1, & if\ 1 \le l \le n - 2s + 2i \\ (2n+3)i - n + l - 1, & if\ n - 2s + 2i + 1 \le l \le n \end{cases}$$

And for $s+1 \le i \le k$,

$$f(y_i^l) = \begin{cases} (2n+3)i - 2n + l - 3, & if\ 1 \le l \le 2i - 2s - 2 \\ (2n+3)i - 2n + l - 2, & if\ 2i - 2s - 1 \le l \le n \end{cases}$$

The set of all edge- sums generated by the above formula forms a consecutive integer sequence (2n+3)k+2, (2n+3)k+3,……. , (4n+8)k+1. Therefore by Lemma 1 , f can be extended to reverse super edge-magic labeling and we obtain the magic constant

$$c = v + e - s$$

$$c = [(2n+3)k + 2k + 1] + (2n+5)k - \{(4n+8)k + 1\}$$

$$= 2k$$

For example, here we show the W-tree WT (2, 3).

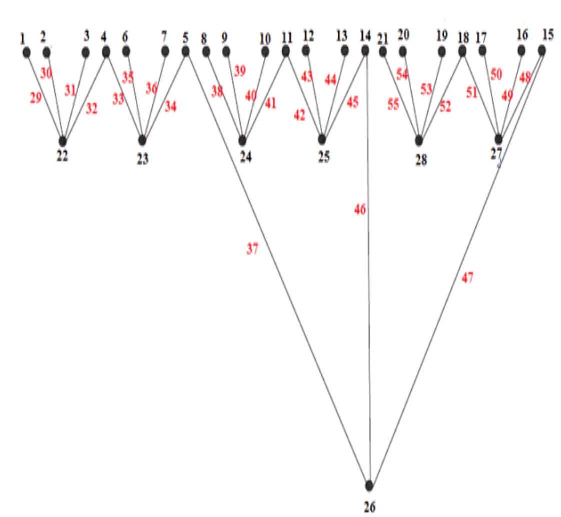

FIG: 4 - WT(2,3)

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