

Efficient BER Analysis of OFDM System over Nakagami-m Fading Channel

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Abstract: In this paper we present an efficient technique for the BER of OFDM system over Nakagami-m fading channels, using well known characteristics function based analysis approach. The average BER expressed in terms of the higher transcendental function such as the confluent hyper geometric functions. Our numerical results show that depending on the number of channel taps, the BER performance may degrade with increasing values of Nakagami-m fading parameters. Phase noise causes significant degradation of performance of the system. The effects of the phase noise on the performance of the OFDM systems have been analytically evaluated. In this project the BER and MSE performance of the OFDM system over multipath fading channel was analysed.

Key Words- Bit Error Rate (BER), Orthogonal Frequency division multiplexing (OFDM) system, Phase noise, RBF (Radial Basis Function) Network, signal to noise ratio (SNR).

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a favoured transmission scheme for many RF/microwave communications systems. It is both efficient and robust, even within a signal environment laden with interference and multipath signals, and is readily scalable in terms of number of users and bandwidth. Given the capabilities of modern application-specific integrated circuits (ASICs) and field-programmable gate arrays (FPGAs), the digital signal processing (DSP) needed to make OFDM work is not a barrier. OFDM is of great interest by researchers and research laboratories all over the world. It has already been accepted for the new wireless local area network standards IEEE 802.11a, High Performance LAN type 2 (HIPERLAN/2) and Mobile Multimedia Access Communication (MMAC) Systems. The analog technologies required for OFDM radios can still pose serious design challenges, especially when application

requirements call for large numbers of OFDM radios within small enclosures at low cost and power consumption. Analog approaches cannot be upgraded as readily or quickly as digital technologies and, as a result, many basic performance parameters still present complex engineering tradeoffs. One of the more critical analog performance parameters, for example, is phase noise. It is especially relevant in OFDM radios due to the large number of closely spaced subcarriers. These subcarriers overlap in the frequency domain, with spectral peaks and nulls arranged to maintain orthogonality. In digital transmission, the number of bit errors is the number of received bits of a data stream over a communication channel that have been altered due to noise, interference, distortion or desynchronization errors. The bit error rate or bit error ratio (BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unitless performance measure, often expressed as a percentage. Soft-

Computing is a collection of techniques spanning many fields that fall under various categories in Computational Intelligence. SoftComputing has three main branches: Fuzzy Logic, genetic algorithm, and Neural Networks. Fuzzy Logic is Soft Computing technique necessary for analyzing complex systems, especially where the data structure is characterized by several linguistic parameters. Genetic algorithm is based upon the concept of evolution and "survival of the fittest". A neural network can perform tasks that cannot have a linear program. When an element of the neural network fails, it can continue without any problem by their parallel nature. A radial basis function (RBF) network is a software system that can classify data and make predictions. RBF networks have some superficial similarities to neural networks, but are actually quite different. An RBF network accepts one or more numeric inputs and generates one or more numeric outputs.

II. SYSTEM MODEL

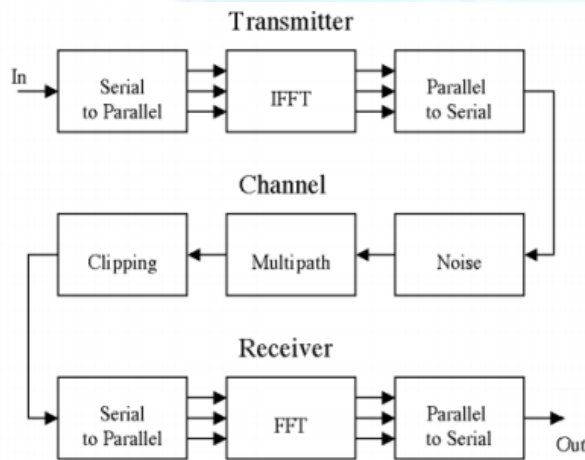


Fig.1 Block diagram of general OFDM system

Consider an OFDM system with N subcarriers. Let $X(k)$ is the k th OFDM data block to be transmitted with N subcarriers. These data are used to modulate N orthogonal sub carriers. Then IDFT is used to modulate the input signal. After modulation signal can be represented as:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp\left(\frac{j2\pi kn}{N}\right) \quad n=0, 1, \dots, N-1$$

According to the nature of the interaction between the wave and obstacle the signal can be reflected

diffracted or diffused. This phenomenon referred to as multipath propagation. The channel impulse response of a multipath fading channel is modeled as a $() () j n h n e \theta -$. In our analysis we assume that frequency synchronization are achieved at the receiver side $r(n)$ can be represented in frequency domain as:

$$R(k) = X(k) * H(k) + N$$

Where N is an independent identically distributed (i.i.d) complex Gaussian noise component with zero mean and unit variance. The amplitude of $H(k)$ is modeled as a Nakagami RV with PDF.

The transmitter first converts the input data from a serial stream to parallel sets. Each set of data contains one symbol, S_i , for each subcarrier. For example, a set of four data would be $[S_0 S_1 S_2 S_3]$. Signal generators perform multiple roles in OFDM system design, and optimizing their use speeds the design process. Modern signal generators can produce modulated and fully coded signals to test receivers, as well as continuous-wave (CW) signals to substitute for frequency references and synthesizers. In both cases, they provide the biggest benefit when they can generate both ideal signals and those which have specific imperfections. Then, the parallel to serial block creates the OFDM signal by sequentially outputting the time domain samples. The channel simulation will allow examination of the effects of noise, multipath, and clipping. By adding random data to the transmitted signal, simple noise can be simulated. Multipath simulation involves adding attenuated and delayed copies of the transmitted signal to the original. This simulates the problem in wireless communication when the signal propagates on many paths. For example, a receiver may see a signal via a direct path as well as a path that bounces off a building. Finally, clipping simulates the problem of amplifier saturation. This addresses a practical implementation problem in OFDM where the peak to average power ratio is high. The receiver performs the inverse of the transmitter. First, the OFDM data are split from a serial stream into parallel sets. The Fast Fourier Transform (FFT) converts the time domain samples back into a frequency domain representation. The magnitudes

of the frequency components correspond to the original data. Finally, the parallel to serial block converts this parallel data into a serial stream to recover the original input data.

Mathematical Description:

In order to do a Monte carlo simulation of an OFDM system, required amount of channel noise has to be generated that is representative of required Eb/N0. In Matlab it is easier to generate a Gaussian noise with zero mean and unit variance. The generated zero-mean-unit-variance noise has to be scaled accordingly to represent the required Eb/N0 or Es/N0. If we have Es/N0, the required noise can be generated from zero-mean-unit-variance-noise by, Since the OFDM system transmits and received the data in symbols, it is appropriate/easier to generate required noise based on Es/N0 instead of Eb/N0 .To convert given Eb/N0 to Es/N0 for an OFDM system, normally for a simple BPSK system, bit energy and symbol energy are same. So Eb/N0 and Es/N0 are same for a BPSK system. But for a OFDM BPSK system, they are not the same. This is because, each OFDM symbol contains additional overhead in both time domain and frequency domain. In the time domain, the cyclic prefix is an additional overhead added to each OFDM symbol that is being transmitted. In the frequency domain, not all the subcarriers are utilized for transmitted the actual data bits, rather a few subcarriers are unused and are reserved as guard bands.

To decrease receiver complexity, every OFDM block is extended, copying in front of it its own end (cyclic prefix). Cyclic prefix is required to operate single frequency networks, where there may exist an ineliminable interference coming from several sites transmitting the same program on the same carrier frequency.

PDF of Nakagami-m RV

In this section, we derive a closed form expression for the first-order PDF and for that we use characteristics function approach (CHF). Let we have an auxiliary function $Z=R \cos(\theta)$. Where R is the fading amplitude and (θ) is the random phase distributed uniformly over $[0, 2\pi]$. The Nakagami-m distribution is given by [5]:

$$f(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mr^2}{\Omega}}, r \geq 0$$

The mathematical definition for finding CHF of X is given as

$$\Psi_X(\eta) = E[\exp(j\eta X)] \quad (4)$$

$$\Psi_X(\eta) = \int_{-\infty}^{\infty} f_X(x) e^{j\eta x} dx$$

The standard derivation for the characteristic function, as derived in is given by:

$$\Psi_{X_n} = \int_0^{\infty} f(r) J_0(\eta r) dr$$

Where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind [18]. Using the eq. (3), eq. (6) can be written as:

$$\Psi_{X_n}(\eta) = \int_0^{\infty} \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mr^2}{\Omega}} J_0(\eta r) dr$$

To simplify the eq. (7) we use integral identity [17, eq. 6.631.1] and eq. (7) can be expressed as:

To obtain the PDF for X the inverse Fourier Transform is applied to get the equation as given in

$$f_X(x) = \frac{1}{2\pi} \int_0^{\infty} e^{-j\eta x} \Psi(\eta) d(\eta)$$

$$f_X(x) = \frac{1}{\pi} \int_0^{\infty} \cos(\eta x) \Psi(\eta) d(\eta)$$

$$f_X(x) = \frac{1}{\pi} \int_0^{\infty} \prod_{n=1}^{M-1} F_1\left(m_n; 1; \frac{-\Omega_n \eta^2}{m_n}\right) \cos(\eta x) d\eta$$

III. STRUCTURAL MODELING

Multipath Fading Channel Multipath fading is a common phenomenon in wireless signal transmission. When a signal is transmitted over a radio channel, it is subject to reflection, refraction and diffraction. Especially in the urban and sub urban areas where cellular phones are most often used, the communication environment changes quickly and thus introduces more complexities and uncertainties to the channel response. This simulator offers a better understanding of this

phenomenon. The channel multipath fading is represented by a randomly time-varying linear filter whose impulse response is limited to some multipath time spread T_0 . The effect of this filter on the input over the given band can be represented as a complex, time varying, tapped-delay line filter with L complex taps at intervals of $1/WL$. must be at least WT_0 because of the effective band limiting of the filter impulse response, but the exact value of L is noncritical in the arguments to follow. Let $F_{i,j}$ be the j th tap of the filter at discrete output time i . Thus, the signal, corrupted by the multipath fading but before the addition of noise, is given at time i by

$$U_i = \sum_{m=0}^{L-1} X_i - m F_{i,m}$$

We denote $(F_{i,0}, F_{i,1}, \dots, F_{i,L-1})^T$ as a random vector F_i

The sample value of this vector is called the channel state at time i . We assume that the vector stochastic process $\dots, F_0, F_1, F_i, \dots$ is zero mean, stationary, and complex Gaussian. Fading is a rapid fluctuation of received signal strength over short time intervals and/or travel distances caused by interference from multiple copies of Tx signal arriving @ Rx at slightly different times. It has two most important effects: 1. Rapid changes in signal strengths over small travel distances or short time periods. 2. Changes in the frequency of signals. Multiple signals arriving a different times. When added together at the antenna, signals are spread out in time. This can cause a smearing of the signal and interference between bits that are received.

Slow fading arises when the coherence time of the channel is large relative to the delay constraint of the channel. In this regime, the amplitude and phase change imposed by the channel can be considered roughly constant over the period of use. Slow fading can be caused by events such as shadowing, where a large obstruction such as a hill or large building obscures the main signal path between the transmitter and the receiver. The received power change caused by shadowing is often model using a log-normal distribution with a standard deviation according to the log-distance path loss model.

Fast fading occurs when the coherence time of the channel is small relative to the delay constraint of

the channel. In this regime, the amplitude and phase change imposed by the channel varies considerably over the period of use.

IV. BER PERFORMANCE ANALYSIS

The conditional BER of a particular modulation is given by $Q(Sx)$, where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$$

Using the alternative representation of above equation as given in

$$Q(Sx) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} e^{-\frac{S^2 x^2}{2 \sin^2(\phi)}} d\phi$$

The error rate denoted by $P(S)$ can be expressed as:

$$P(S) = \int_0^\infty Q(Sx) f_x(x) dx \tag{14}$$

$$P(S) = \int_0^\infty \left(\frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{S^2 x^2}{2 \sin^2(\phi)}\right) d\phi \right) \left(\frac{1}{\pi} \int_0^\infty \cos(\eta x) \Psi(\eta) d\eta \right) dx \tag{15}$$

$$P(S) = \frac{1}{\pi^2} \int_0^\infty \Psi(\eta) d\eta \left(\int_0^{\pi/2} \int_0^\infty \cos(\eta x) \exp\left(-\frac{S^2 x^2}{2 \sin^2(\phi)}\right) dx d\phi \right)$$

V. RESULTS & DISCUSSIONS

In this section the BER performance of an OFDM system over Nakagami- m fading channel is analytically evaluated. By varying the fading parameter m obtaining the BER vs SNR is plotted

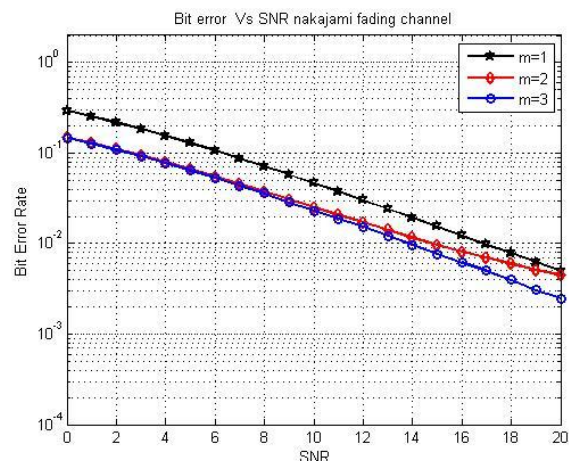


Figure.2 BER Vs SNR for OFDM-BFSK system with single tap

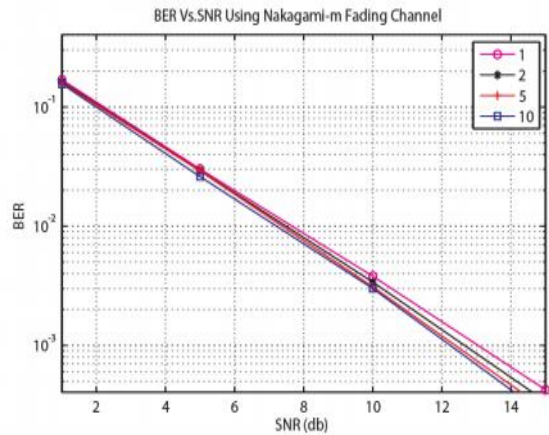


Figure.3 BER Vs SNR for OFDM-BPSK system with single tap

As increasing the fading parameter m , BER starts decreasing as expected for single tap. This fact is already reported in many research papers for Nakagami- m fading channels [10], [11] and [15]. When we consider two tap Nakagami- m channel, slopes of the error rate performance increases if fading parameter m increases. Further, if we increase m , no reduction in BER has been reported rather it starts increasing. So this put a limit to increase the value of m beyond the certain value. This interesting fact is already reported by [22, 23] for the sum of two RVs frequency selective Nakagami- m fading channel and for the frequency non-selective Nakagami- m fading channel respectively. Here we prove this fact for the one RV frequency selective Nakagami- m fading channel and the threshold value for Nakagami- m is achieved to be 1.4 through analytical results.

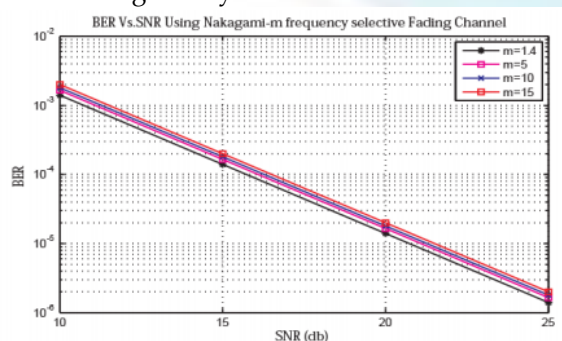


Figure.4 BER Vs SNR for OFDM-BFSK system with two tap

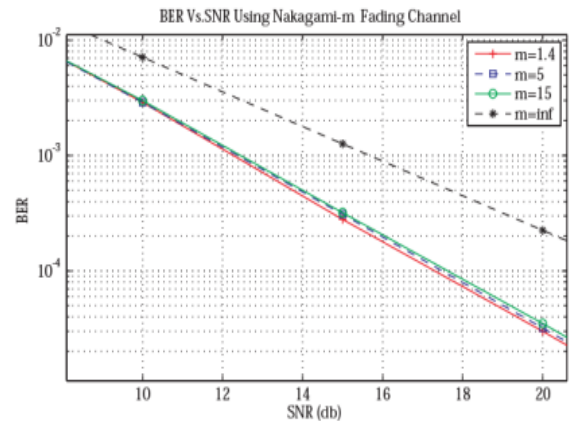


Figure.5 BER Vs SNR for OFDM-BPSK system with two tap.

VI.CONCLUSION

In this paper, a closed form integral PDF has been derived using CHF approach and further utilized for evaluating the closed form expression for average error rate performance of OFDM system over Nakagami- m fading channel with the fading severity index. Based on this unified approach, either numerically or analytically, performance of various modulations over multipath fading channels can be evaluated. Finally it has been found that, depending on the number of channel taps, larger Nakagami- m fading parameters do not necessarily give smaller error rates.

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